Introduction to filters

ACTIVE AND PASSIVE

By Russell Kincaid

This article will be limited to analog filters built with discrete components. Filters can also be made using digital signal processing (DSP) and analog distributed components (transmission lines and resonant cavities).

It is necessary to have an understanding of these terms:

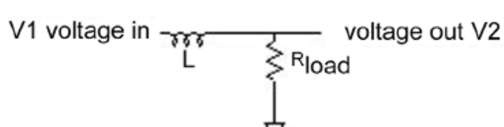
REACTANCE ■ The resistance (impedance) to AC current flow caused by an inductor or capacitor. It is measured in Ohms. The reactance of an inductor increases with frequency; the reactance of a capacitor decreases with frequency. The symbol for inductive reactance is Xl; the symbol for capacitive reactance is Xc.

DECIBEL ■ This is merely the logarithmic compression of data, abbreviated dB. For voltage, the equation is 20*log (V1/V2). For power, the equation is 10*log (P1/P2). The advantage of using dB is that a plot of attenuation versus log frequency will, for the most part, be a straight line. –6 dB represents an attenuation of 0.5, if you work it out, and +6 dB represents a gain of 2. Every –6 dB is 1/2, so –36 dB is .5*.5*.5*.5*.5*.5 = 0.034 and –72 dB is .034*.034 = .0012.

The ideal low pass filter will pass all signals below a given frequency (the cutoff frequency) and not pass any signals above the cutoff frequency. A high pass filter does the opposite. The ideal is not achievable, but complex filters can come close.

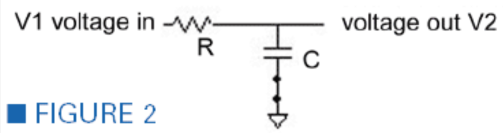
A problem arises when the phase response of the filter is considered. Complex waveforms (pulses) can be considered as composed of many harmonics. If the phase of the component waves is changed relative to one another, the pulse form is changed. In applications where pulse shape is important (television), phase response is an important parameter. The best results are obtained when the phase varies linearly with frequency. Linear phase filters is a subject by itself and is not covered here. As you might expect, sharp cutoff filters have more nonlinearity in the phase response than filters with a more gradual attenuation verses frequency.

The simplest low pass filter consists of a reactive element and a resistor. The reactive element can be an inductor (Figure 1):



■ FIGURE 1

Or a capacitor (Figure 2):

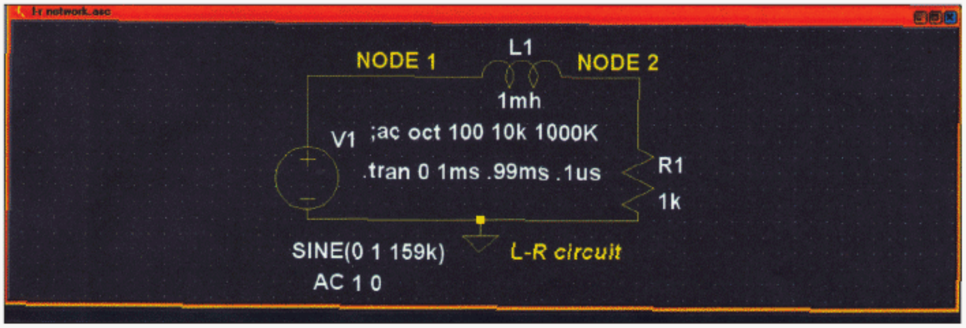


■ FIGURE 2

Figure 1 is an L-R circuit to be analyzed, first is a SPICE transient analysis using a sine wave source of 159 kHz. The reactance of the 1 mH inductor is equal to the 1K ohm resistor at 159 kHz, so we would expect the response to be down 3dB (.707). Figure 2 is a plot of the source (node 1) and the voltage across R1 (node 2). The delay caused by induction in L1 is seen. As the input voltage increases, the current through L1 increases; the increasing current induces a voltage in L1 that opposes the current increase which causes a lag in the current through R1.

After the input voltage passes its peak, the current wants to decrease but the decreasing current induces a voltage in L1 that opposes the decrease; therefore the output current keeps increasing although at a slower rate. There comes a time when the collapsing magnetic field of L1 cannot sustain the output and the voltage at R1 starts to decrease. The same thing happens on the negative half of the sine wave cycle. Since the current and voltage in a resistor is always in phase, it is not necessary, in this case, to show the current separately.

The inductor reactance increases with frequency until it is much larger than the resistor and practically no signal passes through.



■ FIGURE 3

The impedance looking back into the filter is low (the signal source impedance) at low frequencies and approaches R_{load} at high frequencies.

The reactance of the capacitor decreases with frequency until it is low compared to the resistance and the signal is highly attenuated. The impedance looking back into the filter is R + the source impedance and approaches X_c at high frequencies.

For a high pass filter, swap the location of the resistor and reactive element. The same analysis applies but the output voltage increases with frequency.

In both cases, when the reactance is equal to the resistance, the output is 0.707 times the input. It is not 1/2 because the reactive element stores energy during part of the cycle and releases it to the resistor during another part of the cycle. This is called the cutoff frequency and the attenuation is –3 dB.

The equation for reactance of the inductor is: Xl = 2*PI*f*L where PI = 3.14159, f is the frequency in Hz, and L is the inductance in Henries. The reactance of a capacitor is: X_c = 1/(2*PI*f*C) where C is the capacitance in Farads.

If you plot the attenuation in dB of the simple low pass filter versus frequency using semi-log graph paper, the curve approaches a straight line above the cutoff frequency. The slope of the line is –6 dB per octave of frequency. Cascading another stage of filtering will make the slope –12 dB per octave, and so on. The attenuation at the cutoff frequency also increases, being N*-3 dB where N is the number of stages.

The problem in cascading these simple filters is that, in the case of the inductor input filter, the filter impedance at low frequencies is low and difficult to drive. The R-C filter does not have that problem, but the output impedance is high, requiring a very high load impedance so as not to further attenuate the signal.

One way to overcome these problems is to make a filter with only L and C, no resistors except at the source and load. The inductors and capacitors have internal losses that can be represented by resistors, but those losses can be neglected in many cases. We will look at passive L-C filters in the next section.

Another type of filter is the active R-C filter, where feedback is used to overcome the resistive losses. That will be the subject of another article.

The filter introduces a delay in the signal. This can be most easily analyzed using SPICE transient analysis:

Figure 3 is an L-R circuit to be analyzed, first is a SPICE transient analysis using a sine wave source of 159 kHz. The reactance of the 1 mH inductor is equal to the 1K ohm resistor at 159 kHz, so we would expect the response to be down 3 dB (.707). Figure 4 is a plot of the source (node 1) and the voltage across R1 (node 2).

The delay caused by induction in L1 is seen. As the input voltage increases, the current (blue trace) through L1 increases; the increasing current induces a voltage in L1 that opposes the current increase, which causes a lag in the current through R1. After the input voltage passes its peak, the current wants to decrease but the decreasing current induces a voltage in L1 that opposes the decrease; therefore the output current keeps increasing, although at a slower rate.

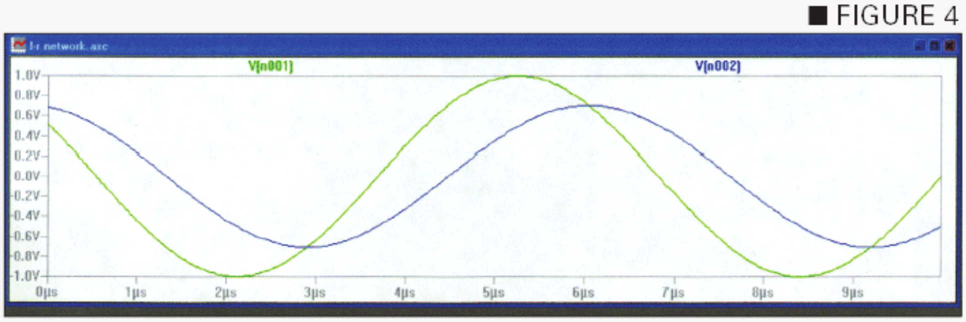
There comes a time when the collapsing magnetic field of L1 cannot sustain the output and the voltage at R1 starts to decrease. The same thing happens on the negative half of the sine wave cycle. Since the current and voltage in a resistor is always in phase, it is not necessary in this case to show the current separately.

Each cycle contains 360 degrees. Measuring the delay of the voltage across R1 as a fraction of the cycle, I get 1.45/11.6 = .125

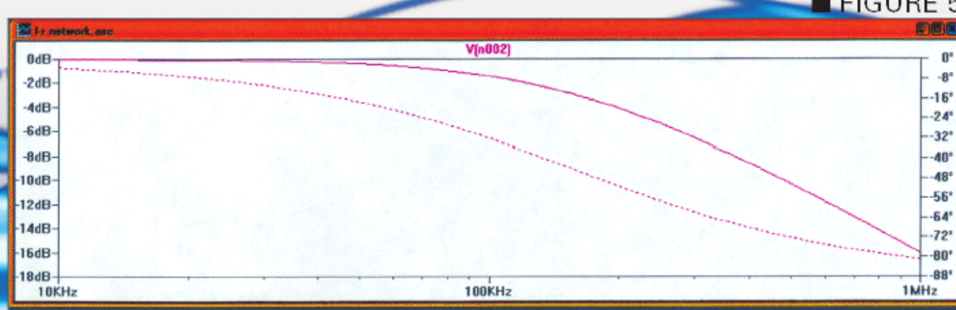
.125*360 = 45 degrees

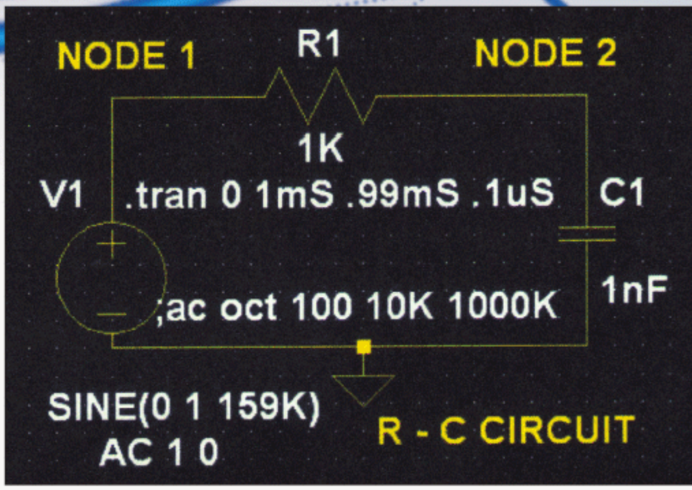
Second, a SPICE frequency sweep (AC) analysis is run from 10 kHz to 1 MHz. The result is in Figure 3. Note that the output at node 1 is nearly equal to the input at 10 kHz because the reactance of L1 is low. At 159 kHz, the output is down 3 dB and

■ FIGURE 4



■ FIGURE 5





■ FIGURE 6

the phase angle is 45 degrees.

As the frequency increases, the reactance of L1 increases causing the output to decrease. The phase angle continues to increase, reaching a limit of 90 degrees at infinite frequency.

Figure 4 is the R-C low pass circuit. The AC analysis in SPICE will be identical to the L-R circuit (Figure 3), because the resistance of R1 is small compared to the reactance of C1 at 10 kHz. As the frequency increases, the reactance of C1 decreases, causing the output (node 2) to decrease. The transient analysis at 159 kHz will be similar but the current waveform, being compared to the voltage across the capacitor, will not be in phase. Theinductor and capacitor are duals in that the voltage across the capacitor is mathematically the same as the current in the inductor, and vice versa.

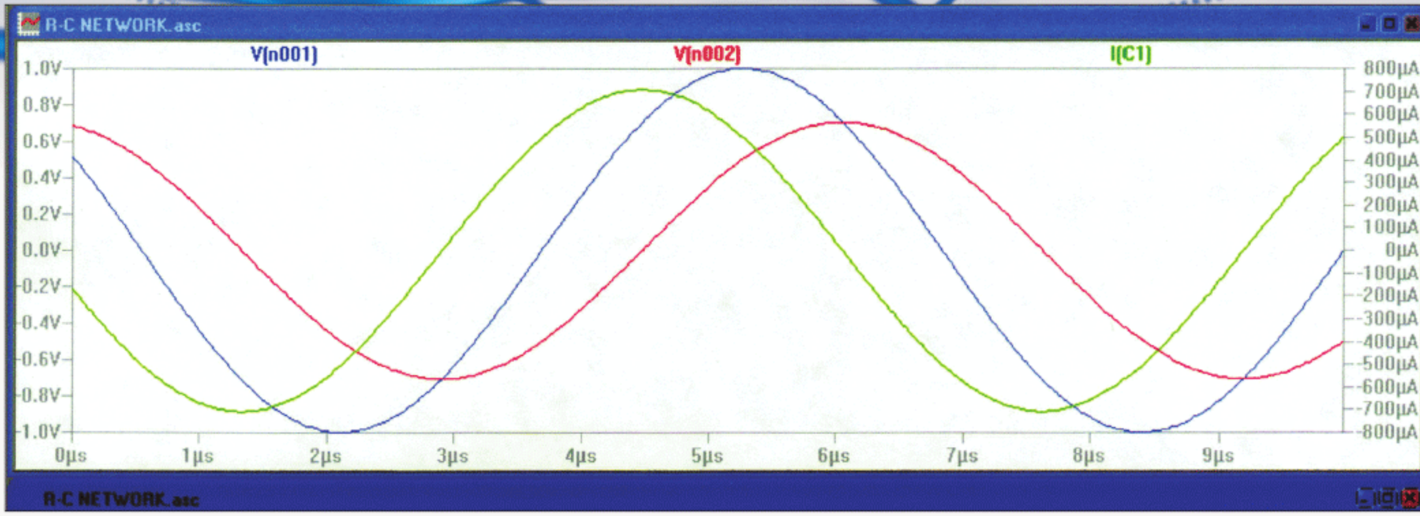
Figure 5 is a plot from the transient analysis of the circuit in Figure 4. The voltage on the capacitor lags the input voltage because it takes time for the current to charge the capacitor. The current responds to the voltage difference between the capacitor charge and the input voltage. The maximum voltage difference is at 45 degrees of the input voltage. Thereafter, the current starts to decrease but since it is still flowing in the same direction, the charge on the capacitor keeps increasing. At 45 degrees past the peak of the input voltage, the input voltage and capacitor voltage will be equal and the current will be zero.

As the current increases in the negative direction, it discharges the capacitor. The analysis of an R-L (high pass) circuit will be the same except current in the capacitor becomes voltage in the inductor and vice versa.

Note that the output at node 1 is nearly equal to the input at 10 kHz because the reactance of L1 is low. At 159 kHz, the output is down 3 dB and the phase angle is 45 degrees. As the frequency increases, the reactance of L1 increases, causing the output to decrease. The phase angle continues to increase, reaching a limit of 90 degrees at infinite frequency.

Figure 6 is the R–C low pass circuit. The AC analysis in SPICE will be identical to the L–R circuit (Figure 3) because the resistance of R1 is small compared to the reactance of C1 at 10 kHz. As the frequency increases, the reactance of C1 decreases, causing the output (node 2) to decrease.

The transient analysis at 159 kHz will be similar, but the current waveform, being



■ FIGURE 7

compared to the voltage across the capacitor, will not be in phase. The inductor and capacitor are duals, in that the voltage across the capacitor is mathematically the same as the current in the inductor, and vice versa.

Figure 7 is a plot from the transient analysis of the circuit in Figure 6. The voltage on the capacitor (red trace) lags the input voltage (blue trace) because it takes time for the current to charge the capacitor. The current (green trace) responds to the voltage difference between the capacitor charge and the input voltage. Note that it leads the voltage on the capacitor by 90 degrees. The maximum voltage difference is at 45 degrees of the input voltage. Thereafter, the current starts to decrease, but since it is still flowing in the same direction, the charge on the capacitor keeps increasing. At 45 degrees past the peak of the input voltage, the input voltage and capacitor voltage will be equal and the current will be zero.

As the current increases in the negative direction, it discharges the capacitor. The analysis of an R–L (high pass) circuit will be the same, except current in the capacitor becomes voltage in the inductor and vice versa. ■